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Unsteady natural convective boundary-layer flow of MHD nanofluid over a stretching surfaces with chemical reaction using the spectral relaxation method: A revised model

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Abstract

We investigate heat and mass transfer in an unsteady MHD nanofluid boundary layer flow due to a stretching surface. The traditional model which here includes the effects of Brownian motion and thermophoresis is revised, so that the nanofluid particle volume fraction on the boundary is passively rather than actively controlled. In this respect the problem is more realistic. This problem is modeled using systems of nonlinear partial differential equations which have been solved numerically using the spectral relaxation method. The results are benchmarked with previously published results.

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Keywords: MHD Nanofluids; chemical reaction parameter; spectral relaxation method.

1. Introduction

The mathematical study over stretching sheet is increasing recent years due it's importance in science and engineering ([1]). The nanofluid represents a liquid in which nanoscale particles are suspended in a base fluid with low thermal conductivity such as water, oils, rthylene glycol etc. In recent years, the concept of nanofluid has been proposed as a route for increasing the performance of heat transfer in liquids. The model for a nanofluid including the effects of Brownian motion and thermophoresis, introduced by Buongiorno [2] which was carried out by Kuznetsov and Nield [3] to the classical problem. In their pioneering problem they employed boundary conditions on the nanoparticle volume fraction. MHD flow, heat and mass transfer has many important technological and industrial applications such as micro MHD pumps, micromixing of physiological samples and drug delivery. Recently, on unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using a spectral relaxation method were reported by Haroun et al. [4]. The aim of the present study is to analyze the effects of Brownian motion and thermophoresis parameters on a boundary condition that is more realistic physically. In a recent paper, Kuznetsov

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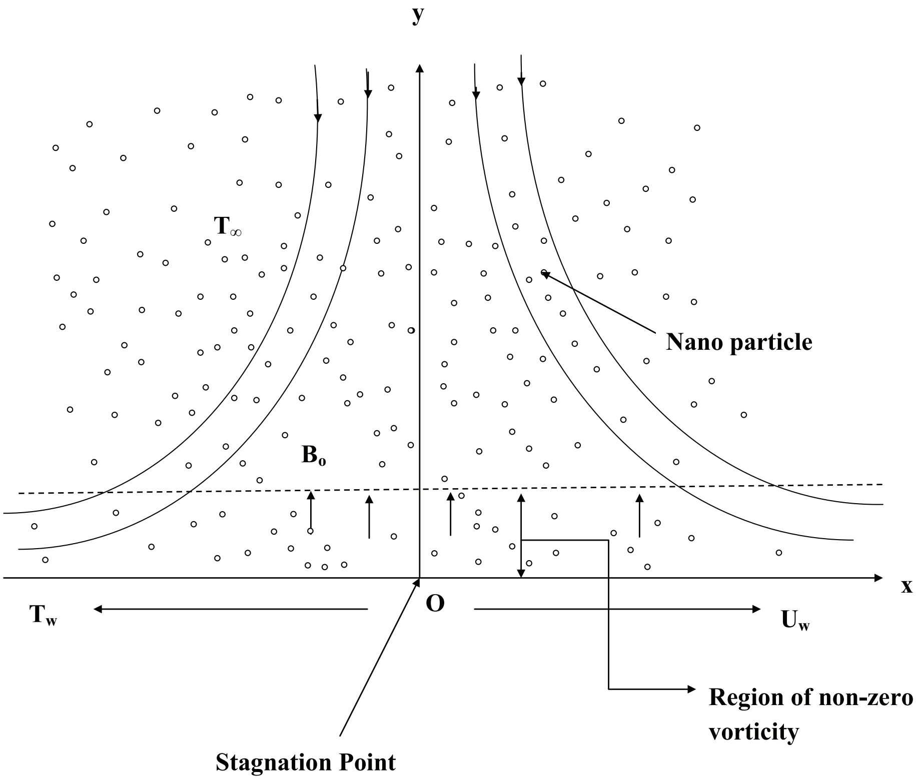


Fig. 1. Geometry of the physical model.

and Nield [5] suggested that the nanoparticle volume fraction flux at the boundary cannot be actively controlled. Very recently, Haroun et al. [12] investigated heat and mass transfer in a magnetohydrodynamic nanofluid flow due to an impulsively started stretching surface. The mathematical problem with this type of boundary conditions has not been studied extensively on previous studies on an unsteady nanofluid in presence of chemical reaction and magnetic field. In this study we solve this type of problem numerically by using spectral relaxation method (Motsa [6]).

2. Governing Equations

Consider the two-dimensional Unsteady natural convective boundary-layer flow of heat and mass transfer nanofluid past a vertical platesituated at $y = 0$ with stretching velocity $u(x) = ax$ where a is a positive constant as shown in Figures 1. At the surface both the nanofluid and the sheet are kept at a constant temperature T_w where $T_w > T_\infty$ is for a heated stretching surface and $T_w < T_\infty$ corresponds to a cooled surface. The boundary layer temperature and nanoparticle volume concentration are T and $\hat{\phi}$ respectively. The ambient fluid temperature and nanoparticle volume fraction are T_∞ and $\hat{\phi}_\infty$ respectively. Using the Boussinesq and the boundary layer approximations, the governing equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) - \frac{\sigma B_0^2}{\rho_{nf}}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \tau^*\left[D_B\frac{\partial\hat{\phi}}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$\frac{\partial\hat{\phi}}{\partial t} + u\frac{\partial\hat{\phi}}{\partial x} + v\frac{\partial\hat{\phi}}{\partial y} = D_B\frac{\partial^2\hat{\phi}}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - K(\hat{\phi} - \hat{\phi}_\infty), \tag{4}$$

where u and v are the fluid velocity and normal velocity components along $x-$ and $y-$directions respectively, μ_{nf}, ρ_{nf}, σ, B_0, g are the the effective dynamic viscosity of the nanofluid, nanofluid density, electrical conductivity, the uniform magnetic field in the y-direction and gravitational acceleration, β_T, T, $\hat{\phi}$, α_{nf}, $\tau^*(= (\rho c)_p/(\rho c)_f)$ are the volumetric thermal expansion coefficient, volumetric solutal expansion coefficient, temperature of fluid in the boundary layer, nanoparticle volume fraction, the thermal diffusivity of the nanofluid, the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid, D_B, D_T, T_∞, K are the Brownian motion coefficient, the thermophoretic diffusion coefficient, mean fluid temperature and the chemical reaction parameter.
The boundary conditions

$$t \geq 0\ : u = U_w(x) = ax,\ v = 0,\ T = T_{\mathrm{w}},\ D_B\frac{\partial\hat{\phi}}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0,$$

$$t \geq 0\ : u, v \to 0,\ T \to T_\infty,\ \hat{\phi} = \hat{\phi}_\infty, \quad \text{as} \quad y \to \infty, \tag{5}$$

where a is the stretching/shrinking rate and stagnation flow rate parameters, with $a < 0$ for shrinking, $a > 0$ for a stretching. The effective dynamic viscosity of the nanofluid was given by Brinkman [7] as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{6}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}},$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + k_f) - 2\phi(k_f - k_s)}{(k_s + k_f) + \phi(k_f - k_s)}, \tag{7}$$

where ϕ and μ_f are the solid volume fraction of nanoparticles and the dynamic viscosity of the base fluid. In equations (1) to (4) and ν_{nf}, ρ_{nf}, $(\rho cp)_{nf}$, k_{nf}, k_f, k_s, ρ_s, $(\rho c_p)_f$, $(\rho c_p)_s$ are the nanofluid kinematic viscosity, the density of nanofluid, the nanofluid heat capacitance, thermal conductivity of the nanofluid, thermal conductivity of the fluid, the thermal conductivity of the solid fractions, the density of the solid fractions, the heat capacity of base fluid, the effective heat capacity of nanoparticles, respectively.
The continuity equation is satisfied by introducing a stream function $\psi(x, y)$ and the following non-dimensional variables, (see Liao [8]) such that

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}. \tag{8}$$

$$\psi = \left[a\nu_f\xi\right]^{\frac{1}{2}}\ xf(\xi,\eta),\ \xi = 1 - \exp(-\tau), \quad \tau = a\,t,\ \eta = \left[\frac{a}{\nu_f\xi}\right]^{\frac{1}{2}}\ y, \tag{9}$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_{\mathrm{w}} - T_\infty},\ \Phi(\xi,\eta) = \frac{\hat{\phi}}{\hat{\phi}_\infty}, \tag{10}$$

where η, ξ and τ are dimensionless variables and the dimensionless time, $f(\xi,\eta)$ is the dimensionless stream function, $\theta(\xi,\eta)$ is the dimensionless temperature and $\phi(\xi,\eta)$ is the dimensionless solute concentration.
We get the dimensionless governing equationa by using the dimensionless variables

$$f''' + \phi_1\left[(1-\xi)\frac{1}{2}\,\eta f'' + \xi\left(ff'' - f'^2 - Mf' + Gr_t\theta\right)\right] = \phi_1\xi(1-\xi)\frac{\partial f'}{\partial\xi}, \tag{11}$$

$$\theta'' + \phi_2 Pr\left(\frac{k_f}{k_{nf}}\right)\left[(1-\xi)\frac{1}{2}\,\eta\theta' + \xi f\theta' + N_b\theta'\Phi' + N_t\theta'^2\right] = \phi_2 Pr\left(\frac{k_f}{k_{nf}}\right)\xi(1-\xi)\frac{\partial\theta}{\partial\xi}, \tag{12}$$

$$\Phi'' + Sc\left[(1-\xi)\frac{1}{2}\,\eta\Phi' + \xi f\Phi'\right] + \frac{N_t}{N_b}\theta'' - \gamma\xi Sc\Phi = Sc\xi(1-\xi)\frac{\partial\Phi}{\partial\xi}, \tag{13}$$

subject to the boundary conditions

$$f(\xi,0)=0,\ f'(\xi,0)=1,\ \theta(\xi,0)=1,\ N_b\Phi'+N_t\theta'=0,\ \eta=0\ ,\quad \xi\geq 0,$$
$$f'(\xi,\infty)=0,\ \theta(\xi,\infty)=0,\ \Phi(\xi,\infty)=0,\quad \eta\longrightarrow\infty,\quad \xi\geq 0. \tag{14}$$

Where primes denote differentiation with respect to η, $\alpha_f = k_f/(\rho c_p)_f$ and $\nu_f = \mu_f/\rho_f$ are the thermal diffusivity and kinetic viscosity of the base fluid, respectively. Other non-dimensional parameters appearing in equations (11) to (13) are Ha, Gr_t, Pr, N_b, N_t, Sc, and γ denote the Magnetic parameter, local temperature Grashof number, Prandtl number, Brownian motion parameter and thermal phoresis parameter, the Schmidt number and scaled chemical reaction parameter. These parameters are defined mathematically as

$$M=\frac{\sigma B_0^2}{a\rho_{nf}}\ ,Gr_t=\frac{g\beta_T(T_w-T_\infty)x^3}{\nu_f^2},\ Pr=\frac{\nu_f}{\alpha_f},$$
$$N_b=\frac{(\rho_c)_pD_B\Phi_\infty}{\nu_f(\rho_p)_f},\ Sc=\frac{\nu_f}{D_B},\ \gamma=\frac{K}{a},\ N_t=\frac{(\rho_c)_p\ D_T(T_w-T_\infty)}{T_\infty\nu_f(\rho_p)_f}. \tag{15}$$

The nanoparticle volume fraction ϕ_1 and ϕ_2 are defined as

$$\phi_1=(1-\phi)^{2.5}\left[1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right],\quad \phi_2=\left[1-\phi+\phi\frac{(\rho c)_s}{(\rho c)_f}\right]. \tag{16}$$

The skin friction coefficient, local Nusselt number are obtained as

$$C_{fx}=\frac{2\tau_w}{\rho_fU_w^2},Nu_x=\frac{xq_w}{k_f\,(T_w-T_\infty)}. \tag{17}$$

The reader will note that the dimensionless mass flux represented by a Sherwood number Sh_x is now identically zero due to the revised nanofluid model.

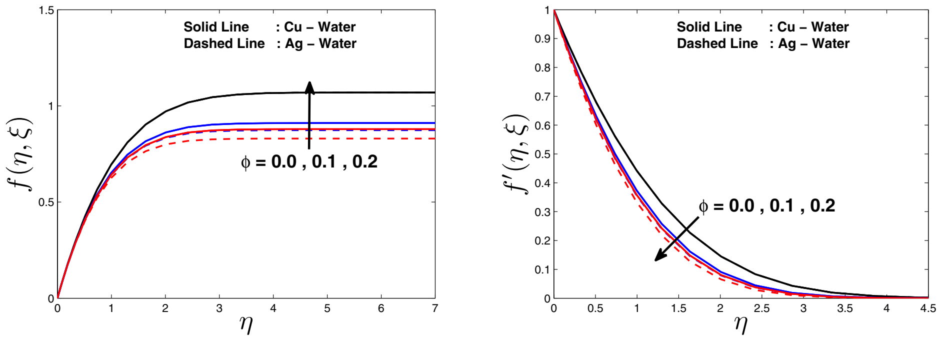
3. Results and Discussion

Table 1. Comparison of the SRM solutions for $f''(\xi,0)$ for different values of ξ when the other parameters are same as those published papers

ξ	Kechil and Hashim [10]	Srinivasa and Eswara [11]	Present Results
0.0	-0.5643740	-0.5643740	-0.5641896
0.1	-0.6150550	-0.6106120	-0.6104676
0.3	-0.7115696	-0.7115610	-0.7115697
0.9	-0.9633761	-0.9623398	-0.9623380
1.0	-1.0000000	-1.0000000	-1.0000019

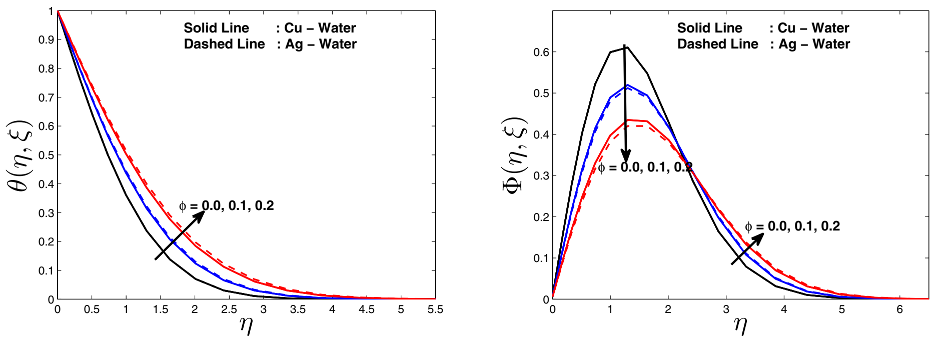
The equations (11) to (13) are solved using the SRM (Motsa [6]). The thermophysical properties of the nanofluids used in the numerical simulations are given in [9]. Extensive calculations have been performed to obtain the velocity, temperature, concentration profiles as well as skin friction etc for various values of physical parameters such as ϕ, M, Gr_t, Pr, N_b, N_t, Sc and γ. To determine the accuracy of our numerical results, the skin friction coefficient is compared with the published results of Kechil and Hashim [10], Srinivasa and Eswara [11] in Tables 1. Here we have varied the ξ while keeping other physical parameters fixed. Table 1 gives a comparison of the SRM results with those obtained by Kechil and Hashim [10], Srinivasa and Eswara [11] when $Gr_t=\gamma=\phi=0$, $Pr=7$, $Sc=0.4$ and $M=0.0$ for different values of the ξ. It is observed that the present results are in good agreement with the previously published results.

The effects of the nanoparticle volume fraction and M on the fluid velocity, temperature, concentration profiles as well as on skin friction are shown in Figures 2 - 4. From Figures 2 - 3, it is evident that the solute concentration decreases with increasing nanoparticle volume fraction while the velocity and temperature skin friction increase. This is because



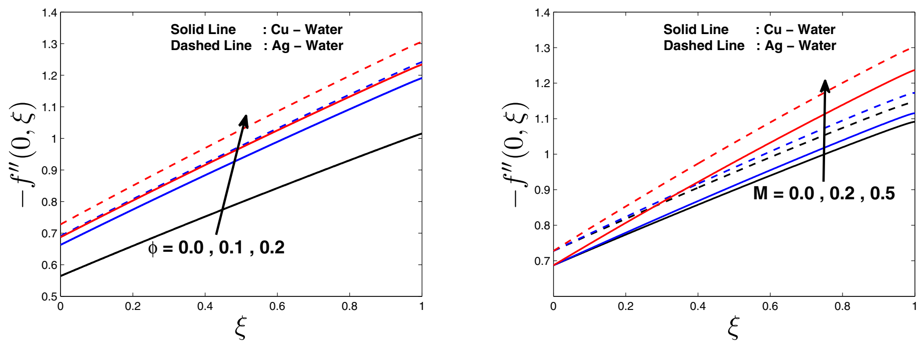
(a) Velocity profiles (b) Velocity profiles

Fig. 2. Effect of various nanoparticle values fraction ϕ on (a) and (b) for $Gr_t = 0.2$, $M = 0.2$, $N_t = 0.01$, $Pr = 7$, $N_b = 0.01$, $\gamma = 1$, $Sc = 1$ and $\xi = 0.5$.



(a) Temperature profiles (b) Concentration profiles

Fig. 3. Effect of various nanoparticle values fraction ϕ on (a) and (b) for $Gr_t = 0.2$, $M = 0.2$, $N_t = 0.01$, $Pr = 7$, $N_b = 0.01$, $\gamma = 1$, $Sc = 1$ and $\xi = 0.5$.



(a) Skin friction coefficient (b) Skin friction coefficient

Fig. 4. Effect of various nanoparticle values fraction ϕ and Magnetic parameter M respectively, on Skin friction coefficient for $Gr_t = 0.2$, $N_t = 0.01$, $Pr = 7$, $N_b = 0.01$, $\gamma = 1$ and $Sc = 1$.

with an increase in nanoparticles volume fraction, the thermal conductivity of the nanofluid increases, which reduces the thermal boundary layer thickness and the temperature gradient at the wall. The axial velocity in the case of an Ag-water nanofluid is comparatively less than that in the case of a Cu-water nanofluid. The temperature distribution in an Ag-water nanofluid is higher than that in a Cu-water nanofluid and this is explained by the observation that the thermal conductivity of silver is less than that of copper. The concentration boundary layer thickness is higher for the case of a Cu-water than that for the case of an Ag-water nanofluid. Figure 4 (a) and (b) show that the skin friction coefficients $-f''(0,\xi)$ increases monotonically with increasing ξ. The result is true for both types of nanofluids. The minimum value of the skin friction in the case of a Cu-water nanofluid is achieved at a smaller value of ξ in comparison with a Ag-water nanofluid. Furthermore, in this paper it is found that the Ag-water nanofluid shows higher drag as compared to the a Cu-water nanofluid. The maximum value of the skin friction in the case of a Ag-water nanofluid is achieved at the value of $\xi = 1$ in comparison with an Cu-water nanofluid in two figures (a) and (b)respectively.

4. Conclusions

We have investigated the heat and mass transfer in an unsteady MHD boundary layer flow in nanofluid due to a stretching surfaces with chemical reaction and an applied magnetic field. From the numerical simulations, some results can be drawn as follow:

[i] The velocity profile increase with increase in the nanoparticle volume fraction while the opposite trend is observed with increase in the value of the nanoparticle volume fraction.

[ii] The temperature and concentration profiles increase with increasing in the values of the nanoparticle volume fraction and thermophoresis parameter while the opposite trend is observed for the concentration profile with increasing in the values of Brownian motion parameter.

[iii] The values of skin friction increase with increase in the values of the nanoparticle volume fraction and magnetic parameter M.

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Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

References

[1] T.R. Mahapatra, S. Mondal, D. Pal, Heat transfer due to magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface in the presence of thermal radiation and suction/injection, ISRN Thermodynamics, 2012, Article ID 465864, doi:10.5402/2012/465864 128 (2006) 240–250.

[2] J. Buongiorno, Convective transport in nanofluids, Journal of Heat Transf. 128 (2006) 240–250.

[3] A.V. Kuznetsov, D.A. Nield, Natural convective boundary layer flow of a nanofluid past a vertical plate, International Journal Thermal Sciences, 49 (2010) 243–247.

[4] A.H. Nageeb, P. Sibanda, S. Mondal, S.S. Motsa, On unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using the spectral relaxation method, Boundary Value Problems, 24 (2015) DOI 10.1186/s13661–015–0289–5.

[5] A.V. Kuznetsov, D.A. Nield, Natural convective boundary layer flow of a nanofluid past a vertical plate: A revised model, International Journal Thermal Sciences, 77 (2014) 126–129.

[6] S.S. Motsa, A New spectral relaxation method for similarity variable nonlinear boundary layer flow systems, Chemical Engineering Communications, 201 (2014) 241–256.

[7] H.C. Brinkman, The viscosity of concentrated suspensions and solution, The Journal of Chemical Physics, 20 (1952) 571.

[8] S.J. Liao, An analytic solution of unsteady boundary layer flows caused by an impulsively stretching plate, Communications in Nonlinear Science and Numerical Simulation, 11 (2006) 326–329.

[9] M. Sheikholeslami, M.G. Bandpy, D.D. Ganji, S. Soleimani, S.M. Seyyedi, Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field, International Communications in Heat and Mass Transf. 39 (2012) 1435–1443.

[10] S.A. Kechil, I. Hashim, Series solution for unsteady boundary-layer flows due to impulsively stretching plate, Chinese Physics Letters, 24 (2007) 139–142.

[11] A.H. Srinivasa, A.T. Eswara, Unsteady MHD Laminar boundary layer flow due to an impulsively stretching surface, Proceedings of the World Congress on Engineering, WCE, July 6-8, London, U.K. 2011.

[12] A.H. Nageeb, P. Sibanda, S. Mondal, S.S. Motsa, M.M. Rashidi, Heat and mass transfer of nanofluid through an impulsively vertical stretching surface using the spectral relaxation method, Boundary Value Problems, 2015 (2015) 161, DOI 10.1186/s13661-015-0424-3.